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In $\triangle ABC$ the following relationship holds:

$$(\csc A)^{\csc A} (\csc B)^{\csc B} (\csc C)^{\csc C} \geq \left(\frac{2}{\sqrt{3}}\right)^{2\sqrt{3}}$$

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Solution by Tapas Das-India

$$\sum \csc A = \sum \frac{1}{\sin A} \stackrel{\text{Bergstrom}}{\geq} \frac{9}{\sum \sin A} \stackrel{\text{Jensen}}{\geq} \frac{9}{3 \cdot \sin \frac{\pi}{3}} = 2\sqrt{3} \quad (1)$$

$$(\csc A)^{\csc A} \cdot (\csc B)^{\csc B} \cdot (\csc C)^{\csc C} \stackrel{GM-HM}{\geq} \left(\frac{\csc A + \csc B + \csc C}{\sum \frac{\csc A}{\csc A}} \right)^{\sum \csc A} =$$

$$= \left(\frac{1}{3} \sum \csc A \right)^{\sum \csc A} \stackrel{(1)}{\geq} \left(\frac{1}{3} \cdot 2\sqrt{3} \right)^{2\sqrt{3}} = \left(\frac{2}{\sqrt{3}} \right)^{2\sqrt{3}}$$

Equality holds for an equilateral triangle.