ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{ab}{r_c^2}+\frac{bc}{r_a^2}+\frac{ca}{r_b^2}\geq 4$$

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Solution by Tapas Das-India

$$h_a=\frac{bc}{2R}\ then\ bc=2R\,h_a\,,similarly, ca=2R\,h_b, ab=2R\,h_c$$

$$\frac{ab}{r_c^2}+\frac{bc}{r_a^2}+\frac{ca}{r_b^2}=\frac{2Rh_c}{r_c^2}+\frac{2Rh_a}{r_a^2}+\frac{2Rh_b}{r_b^2}=2R\left(\frac{h_c}{r_c^2}+\frac{h_a}{r_a^2}+\frac{h_b}{r_b^2}\right)=$$

$$=2R\left(\frac{\frac{1}{r_c^2}}{\frac{1}{h_c}}+\frac{\frac{1}{r_a^2}}{\frac{1}{h_a}}+\frac{\frac{1}{r_b^2}}{\frac{1}{h_b}}\right)\overset{Bergstom}{\geq}2R\,\frac{\left(\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c}\right)^2}{\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}}=2R\,\frac{\left(\frac{1}{r}\right)^2}{\frac{1}{r}}=\frac{2R}{r}\overset{Euler}{\geq}2\times 2=4$$

Equality holds for an equilateral triangle.