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In $\triangle ABC$ the following relationship holds:

$$\left(\sec \frac{A}{2}\right)^{\sec \frac{A}{2}} \left(\sec \frac{B}{2}\right)^{\sec \frac{B}{2}} \left(\sec \frac{C}{2}\right)^{\sec \frac{C}{2}} \geq \left(\frac{2}{\sqrt{3}}\right)^{2\sqrt{3}}$$

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Solution by Tapas Das-India

$$\sum \sec \frac{A}{2} \stackrel{\text{Jensen}}{\geq} 3 \sec \left(\frac{A+B+C}{6}\right) = 3 \sec \frac{\pi}{6} = 2\sqrt{3} \quad (1)$$

$$\begin{aligned} \left(\sec \frac{A}{2}\right)^{\sec \frac{A}{2}} \left(\sec \frac{B}{2}\right)^{\sec \frac{B}{2}} \left(\sec \frac{C}{2}\right)^{\sec \frac{C}{2}} &\stackrel{\text{Weighted HM} \leq \text{GM}}{\geq} \\ &\geq \left(\frac{\sum \sec \frac{A}{2}}{\sum \frac{\sec \frac{A}{2}}{\sec \frac{A}{2}}}\right)^{\sum \sec \frac{A}{2}} \stackrel{(1)}{\geq} \left(\frac{2\sqrt{3}}{3}\right)^{2\sqrt{3}} = \left(\frac{2}{\sqrt{3}}\right)^{2\sqrt{3}} \end{aligned}$$

Equality holds for an equilateral triangle.