

ROMANIAN MATHEMATICAL MAGAZINE

In acute $\triangle ABC$ the following relationship holds:

$$(\sec A)^{\sec A} (\sec B)^{\sec B} (\sec C)^{\sec C} \geq 64$$

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Solution by Tapas Das-India

$$\sum \cos A = 1 + \frac{r}{R} \stackrel{\text{Euler}}{\leq} \frac{3}{2} \quad (1)$$

$$\sum \sec A = \sum \frac{1}{\cos A} \stackrel{\text{Bergstrom}}{\geq} \frac{(1+1+1)^2}{\sum \cos A} \stackrel{(1)}{\geq} \frac{9}{\frac{3}{2}} = 6 \quad (2)$$

$$(\sec A)^{\sec A} (\sec B)^{\sec B} (\sec C)^{\sec C} \stackrel{\text{associated weight HM} \leq \text{GM}}{\geq} \left(\frac{\sum \sec A}{\sum \frac{\sec A}{\sec A}} \right)^{\sum \sec A} \stackrel{(2)}{\geq} \left(\frac{6}{3} \right)^6 = 64$$

Equality holds for an equilateral triangle.