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In ΔABC the following relationship holds:

$$\left(\sin\frac{A}{2}+\sin\frac{B}{2}\right)^{\sin\frac{C}{2}}\left(\sin\frac{C}{2}+\sin\frac{B}{2}\right)^{\sin\frac{A}{2}}\left(\sin\frac{A}{2}+\sin\frac{C}{2}\right)^{\sin\frac{B}{2}}\leq 1$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$\left(\sin\frac{A}{2}+\sin\frac{B}{2}\right)^{\sin\frac{C}{2}}\left(\sin\frac{C}{2}+\sin\frac{B}{2}\right)^{\sin\frac{A}{2}}\left(\sin\frac{A}{2}+\sin\frac{C}{2}\right)^{\sin\frac{B}{2}}\overset{AM-GM}{\overbrace{\leq}}$$

$$\leq\left(\frac{\sum_{cyc}\sin\frac{A}{2}\left(\sin\frac{C}{2}+\sin\frac{B}{2}\right)}{\sum_{cyc}\sin\frac{A}{2}}\right)^{\sum_{cyc}\sin\frac{A}{2}}=\left(\frac{2.\sum_{cyc}\sin\frac{A}{2}.\sin\frac{B}{2}}{\sum_{cyc}\sin\frac{A}{2}}\right)^{\sum_{cyc}\sin\frac{A}{2}}\leq$$

$$\leq\left(\frac{\frac{2}{3}\cdot\left(\sum_{cyc}\sin\frac{A}{2}\right)^2}{\sum_{cyc}\sin\frac{A}{2}}\right)^{\sum_{cyc}\sin\frac{A}{2}}\leq\left(\frac{2}{3}\cdot\sum_{cyc}\sin\frac{A}{2}\right)^{\sum_{cyc}\sin\frac{A}{2}}\overset{Jensen}{\overbrace{\leq}}\left(\frac{2}{3}\cdot\frac{3}{2}\right)^{\frac{3}{2}}=1$$

$$Equality\ \ holds\ \ for: A=B=C.$$