

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\left(\sec \frac{A}{2}\right)^{\tan \frac{A}{2}} \left(\sec \frac{B}{2}\right)^{\tan \frac{B}{2}} \left(\sec \frac{C}{2}\right)^{\tan \frac{C}{2}} \geq \left(\frac{2}{\sqrt{3}}\right)^{\sqrt{3}}$$

*Proposed by Nguyen Hung Cuong-Vietnam*

*Solution by Tapas Das-India*

$$\sum \sin \frac{A}{2} \stackrel{\text{Jensen}}{\leq} 3 \sin \left(\frac{A+B+C}{6}\right) \stackrel{A+B+C=\pi}{=} \frac{3}{2} \quad (1)$$

$$\sum \tan \frac{A}{2} = \frac{4R+r}{s} \stackrel{\text{Doucet}}{\geq} \sqrt{3} \quad (2)$$

$$\begin{aligned} & \left(\sec \frac{A}{2}\right)^{\tan \frac{A}{2}} \left(\sec \frac{B}{2}\right)^{\tan \frac{B}{2}} \left(\sec \frac{C}{2}\right)^{\tan \frac{C}{2}} \stackrel{\text{Weighted GM} \geq \text{HM}}{\geq} \\ & \geq \left( \frac{\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2}}{\frac{\tan \frac{A}{2}}{\sec \frac{A}{2}} + \frac{\tan \frac{B}{2}}{\sec \frac{B}{2}} + \frac{\tan \frac{C}{2}}{\sec \frac{C}{2}}} \right)^{\sum \tan \frac{A}{2}} = \left( \frac{\sum \tan \frac{A}{2}}{\sum \sin \frac{A}{2}} \right)^{\sum \tan \frac{A}{2}} \stackrel{(1) \& (2)}{\geq} \left( \frac{\sqrt{3}}{\frac{3}{2}} \right)^{\sqrt{3}} = \left( \frac{2}{\sqrt{3}} \right)^{\sqrt{3}} \end{aligned}$$

*Equality holds for  $A = B = C$ .*