ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(\sin A+\sin B)^{\sin C}(\sin B+\sin C)^{\sin A}(\sin C+\sin A)^{\sin B}\le\left(\sqrt{3}\right)^{\frac{3\sqrt{3}}{2}}$$

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Solution by Tapas Das-India

$$\sin A+\sin B+\sin C=\frac{s}{R}\overset{Mitrinovic}{\le}\frac{3\sqrt{3}R}{2R}=\frac{3\sqrt{3}}{2}\ (1)$$

$$Using\ weighted\ AM\ge GM:$$

$$(\sin A+\sin B)^{\sin C}(\sin B+\sin C)^{\sin A}(\sin C+\sin A)^{\sin B}$$

$$\le\left(\frac{\sum\sin C\,(\sin A+\sin B)}{\sin C+\sin A+\sin B}\right)^{\sin A+\sin B+\sin C}=$$

$$=\left(\frac{2\sum\sin C\sin A}{\sin C+\sin A+\sin B}\right)^{\sin A+\sin B+\sin C}\le\left(\frac{\frac{2(\sin A+\sin B+\sin C)^2}{3}}{\sin C+\sin A+\sin B}\right)^{\sin A+\sin B+\sin C}=$$

$$=\left(\frac{2}{3}(\sin A+\sin B+\sin C)\right)^{\sin A+\sin B+\sin C}\overset{(1)}{\le}\left(\frac{2}{3}\frac{3\sqrt{3}}{2}\right)^{\frac{3\sqrt{3}}{2}}=\left(\sqrt{3}\right)^{\frac{3\sqrt{3}}{2}}$$

Equality holds for an equilateral triangle.