ROMANIAN MATHEMATICAL MAGAZINE

$$In\ acute\ \Delta ABC\ following\ relationship\ \ holds:$$

$$\rho_H \le R - \frac{3r}{2}$$

$$\rho_H: inradius\ of\ orthic\ triangle\ of\ \Delta ABC$$

Proposed by Mehmet Șahin-Turkiye

Solution by Tapas Das-India

$$Let\ \Delta DEF\ is\ the\ orthic\ triangle. The\ points\ D, E, F\ lie\ on\ the$$

$$ninepoint\ circle\ then\ circumradius\ of\ \Delta DEF\ is \frac{R}{2},$$

$$where\ R = \ circumradius\ of\ \Delta ABC$$

$$Inradius(Orthic)\ \rho_H \overset{Euler}{\le} \frac{circumradius(Orthic)}{2}$$

$$\rho_H \le \frac{1}{2}\left(\frac{R}{2}\right) = \frac{R}{4}$$

$$We\ need\ to\ show:$$

$$\rho_H \le R - \frac{3r}{2}\ or\ \frac{R}{4} \le R - \frac{3r}{2}\ or \frac{3R}{4} \ge \frac{3r}{2}\ or\ R \ge 2r\ true\ by\ Euler$$

Equality holds for an equilateral triangle.