ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{\ln^{k+1} a}{\ln(b+c-a)} \geq \frac{\ln^k abc}{3^{k-1}}, k \in N$$

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Solution by Tapas Das-India

$$abc = 4Rrs \overset{Euler}{\geq} 8r^2 s = 8(s-a)(s-b)(s-c) =$$

$$= (2s-2a)(2s-2b)(2s-2c) = (a+b-c)(b+c-a)(c+a-b)\ (1)$$

$$and \sum \ln(b+c-a) = \ln\big((a+b-c)(b+c-a)(c+a-b)\big) \overset{(1)}{\leq} \ln abc\ (\mathrm{A})$$

$$\left(\sum \frac{\ln^{k+1} a}{\ln(b+c-a)}\right) \sum \ln(b+c-a)(1+1+1)^{n-1} \overset{Holder}{\geq}$$

$$\geq \frac{1}{3^{k-1}} \frac{(\ln a + \ln b + \ln c)^{\mathrm{k}+1}}{\ln(b+c-a) + \ln(c+a-b) + \ln(b+a-c)} \overset{(A)}{\geq} \frac{1}{3^{k-1}} \frac{(\ln abc)^{k+1}}{\ln abc} = \frac{\ln^k abc}{3^{k-1}}$$

Equality holds for an equilateral triangle.