ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum \frac{\sin^n A}{1+\cos A} \geq 2\left(\frac{\sqrt{3}r}{R}\right)^n, \quad n \in N$$

Proposed by Marin Chirciu-Romania

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum \frac{\sin^n A}{1+\cos A} \geq 3\left(\frac{\sin^n A}{2\cos^2\frac{A}{2}} \cdot \frac{\sin^n B}{2\cos^2\frac{B}{2}} \cdot \frac{\sin^n C}{2\cos^2\frac{C}{2}}\right)^{\frac{1}{3}} = \frac{3}{2} \cdot \left(\frac{(\sin(A) \cdot \sin(B) \cdot \sin(C))^n}{\left(\cos\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \cos\frac{C}{2}\right)^2}\right)^{\frac{1}{3}} =$$

$$= \frac{3}{2} \cdot \left(\frac{\left(\frac{F}{2R^2}\right)^n}{\left(\frac{S}{4R}\right)^2}\right)^{\frac{1}{3}} = \frac{3}{2} \cdot \left(\frac{\left(\frac{Sr}{2R^2}\right)^n}{\left(\frac{S}{4R}\right)^2}\right)^{\frac{1}{3}} \overset{Mitrinovici}{\geq} \frac{3}{2} \cdot \left(\frac{\left(\frac{3\sqrt{3}r^2}{2R^2}\right)^n}{\left(\frac{3\sqrt{3}R}{8R}\right)^2}\right)^{\frac{1}{3}} =$$

$$= \frac{3}{2} \cdot \frac{4}{3}\left(\left(\frac{\sqrt{3}}{2}\right)^n \cdot \left(\frac{\sqrt{3}r}{R}\right)^{2n}\right)^{\frac{1}{3}} = 2 \cdot \left(\frac{\sqrt{3}}{2}\right)^{\frac{n}{3}} \cdot \left(\frac{\sqrt{3}r}{R}\right)^{\frac{2n}{3}} = 2 \cdot \left(\frac{\sqrt{3}r}{R}\right)^n \cdot \frac{\left(\frac{\sqrt{3}}{2}\right)^{\frac{n}{3}}}{\left(\frac{\sqrt{3}r}{R}\right)^{\frac{n}{3}}} =$$

$$= 2\left(\frac{\sqrt{3}r}{R}\right)^n \cdot \left(\frac{R}{2r}\right)^{\frac{n}{3}} \overset{Euler}{\geq} 2\left(\frac{\sqrt{3}r}{R}\right)^n$$

$$Equality \;\; holds \;\; for: \quad A = B = C$$