

ROMANIAN MATHEMATICAL MAGAZINE

If $x + y + z = \frac{\pi}{2}$ then:

$$\sum \frac{x^n}{\tan y \tan z} \geq 9 \left(\frac{\pi}{6}\right)^n, n \in \mathbb{N}$$

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$$x + y + z = \frac{\pi}{2} \text{ or, } (x + y) = \frac{\pi}{2} - z$$

$$\tan(x + y) = \tan\left(\frac{\pi}{2} - z\right) \text{ or } \frac{\tan x + \tan y}{1 - \tan x \tan y} = \cot z$$

$$\text{or, } \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{1}{\tan z} \text{ or } \sum \tan x \tan y = 1 \quad (1)$$

$$\left(\sum \frac{x^n}{\tan y \tan z}\right) \left(\sum \tan x \tan y\right) (1 + 1 + 1)^{n-2} \stackrel{HOLDER}{\geq} (x + y + z)^n$$

$$\sum \frac{x^n}{\tan y \tan z} \geq \frac{(x + y + z)^n}{3^{n-2} \cdot \sum \tan x \tan y} = \frac{\left(\frac{\pi}{2}\right)^n}{3^{n-2}} = 9 \left(\frac{\pi}{6}\right)^n$$

Equality holds for $x = y = z = \frac{\pi}{6}$.