In ΔABC the following relationship holds:

$$\sum \frac{\sin^2 A}{1 + \cos A} \geq \frac{6r^2}{R^2}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \frac{\sin^2 A}{1 + \cos A} \overset{Bergstrom}{\geq} \frac{(\sum \sin A)^2}{3 + \sum \cos A} = \frac{\left(\frac{s}{R}\right)^2}{3 + \left(1 + \frac{r}{R}\right)} =$$

$$= \frac{s^2}{R^2} \cdot \frac{R}{4R + r} \overset{Mitrinovic\ \&\ Euler}{\geq} \frac{27r^2}{R.\left(4R + \frac{R}{2}\right)} = \frac{6r^2}{R^2}$$

Equality holds for an equilateral triangle.