ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\left(\frac{a}{2b+c}\right)^2+\left(\frac{b}{2c+a}\right)^2+\left(\frac{c}{2a+b}\right)^2\geq\frac{4}{3}\left(\frac{r}{R}\right)^2$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\left(\frac{a}{2b+c}\right)^2+\left(\frac{b}{2c+a}\right)^2+\left(\frac{c}{2a+b}\right)^2\overset{Bergstrom}{\geq}$$

$$\frac{(a+b+c)^2}{5(a^2+b^2+c^2)+4(ab+bc+ac)}\overset{\substack{Leibniz\\Leuenberger\\Mitrinovic}}{\geq}\frac{108r^2}{45R^2+36R^2}$$

$$\boxed{a^2+b^2+c^2\leq 9R^2\ \ (Leibniz)}$$

$$\boxed{ab+bc+ac\leq 9R^2\ \ (Leuenberger)}$$

$$\boxed{s\geq 3\sqrt{3}r\ \ (Mitrinovic)}$$

$$=\frac{108r^2}{81R^2}=\frac{4}{3}\left(\frac{r}{R}\right)^2\ \ (Proved)$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$