In ΔABC the following relationship holds:

$$\sum\left(\frac{r_b r_c}{r_b+r_c}\right)\frac{h_a}{r_a}\geq\frac{9r^2}{R}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum\left(\frac{r_b r_c}{r_b+r_c}\right)\frac{h_a}{r_a}=\sum\frac{r_b r_c h_a}{r_b r_a+r_c r_a}=\sum\frac{s(s-a)\frac{2F}{a}}{s(s-c)+s(s-b)}=$$

$$=\sum\frac{(s-a)\frac{2F}{a}}{(s-c)+(s-b)}=\sum\frac{(s-a).2F}{a^2}=$$

$$=2F\left(\frac{(s-a)}{a^2}+\frac{(s-b)}{b^2}+\frac{(s-c)}{c^2}\right)\overset{Chebyshev}{\geq}$$

$$In\ \Delta ABC\ \ wlog\ \ a\leq b\leq c\rightarrow\boxed{\begin{array}{c}s-a\geq s-b\geq s-c\\ \frac{1}{a^2}\geq\frac{1}{b^2}\geq\frac{1}{c^2}\end{array}}$$

$$\overset{Chebyshev}{\geq}\ 2F.\frac{1}{3}(S-a+S-b+S-c)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)\overset{Steining}{\geq}$$

$$\geq\frac{2}{3}.sr.s.\frac{1}{2Rr}=\frac{s^2}{3R}\overset{Mitrinovic}{\geq}\frac{27r^2}{3R}=\frac{9r^2}{R}$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$