ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\cos^2\left(\frac{A-B}{2}\right)}{\tan\frac{C}{2}}+\frac{\cos^2\left(\frac{B-C}{2}\right)}{\tan\frac{A}{2}}+\frac{\cos^2\left(\frac{C-A}{2}\right)}{\tan\frac{B}{2}}\ge\frac{6\sqrt{3}r}{R}$$

Proposed by George Apostolopoulos-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{\cos^2\left(\frac{A-B}{2}\right)}{\tan\frac{C}{2}}=\frac{\left(\frac{a+b}{c}\right)^2.\sin^2\frac{C}{2}}{\tan\frac{C}{2}}=\frac{1}{2}\left(\frac{a+b}{c}\right)^2.\sin C$$

$$\sum_{cyc}\frac{1}{2}\left(\frac{a+b}{c}\right)^2.\sin C=\frac{1}{2}\sum_{cyc}\left(\frac{a+b}{c}\right)^2.\sin C\overset{A-G}{\ge}$$

$$\ge\frac{3}{2}\left(\left(\frac{a+b}{c}.\frac{b+c}{a}.\frac{c+a}{b}\right)^2.\sin A.\sin B.\sin C\right)^{\frac{1}{3}}\overset{A-G}{\ge}\frac{3}{2}.\left(\left(\frac{2\sqrt{ab}}{c}.\frac{2\sqrt{bc}}{a}.\frac{2\sqrt{ac}}{b}\right)^2.\frac{F}{2R^2}\right)^{\frac{1}{3}}$$

$$=6\left(\frac{F}{2R^2}\right)^{\frac{1}{3}}=6\left(\frac{Sr}{2R^2}\right)^{\frac{1}{3}}\overset{Mitrinovici}{\ge}6\left(\frac{3\sqrt{3}r^2}{2R^2}\right)^{\frac{1}{3}}=6\sqrt{3}\left(\frac{r^2}{2R^2}\right)^{\frac{1}{3}}\overset{R\ge 2r\ (Euler)}{\ge}6\sqrt{3}.\frac{r}{R}$$

$$Equality\ \ holds\ for:\ A=B=C$$