ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a^2}{h_a^2 m_a^2}+\frac{r_b^2}{h_b^2 m_b^2}+\frac{r_c^2}{h_c^2 m_c^2}\ge\frac{4}{3R^2}$$

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Solution by Tapas Das-India

$$WLOG\ a\ge b\ge c\ then\ r_a\ge r_b\ge r_c\ and\ h_a\le h_b\le h_c$$

$$\frac{r_a^2}{h_a^2 m_a^2}+\frac{r_b^2}{h_b^2 m_b^2}+\frac{r_c^2}{h_c^2 m_c^2}=\sum\frac{r_a^2}{h_a^2 m_a^2}\overset{Panaitopol}{\ge}\sum\frac{r_a^2}{h_a^2\left(\frac{Rh_a}{2r}\right)^2}=$$

$$=\frac{4r^2}{R^2}\sum\frac{r_a^2}{h_a^4}\overset{Chebyshev}{\ge}\frac{4r^2}{R^2}\cdot\frac{1}{3}\sum r_a^2\sum\frac{1}{h_a^4}=\frac{4r^2}{R^2}\cdot\frac{1}{3}\sum\frac{1^3}{\left(\frac{1}{r_a}\right)^2}\sum\left(\frac{1}{h_a}\right)^4\overset{Radon\ \&\ CBS}{\ge}$$

$$\ge\frac{4r^2}{R^2}\cdot\frac{1}{3}\cdot\frac{27}{\left(\sum\frac{1}{r_a}\right)^2}\cdot\frac{1}{27}\left(\sum\frac{1}{h_a}\right)^4=\frac{4r^2}{R^2}\cdot\frac{1}{3}27r^2\cdot\frac{1}{r^4}\cdot\frac{1}{27}=\frac{4}{3R^2}$$

Equality holds for an equilateral triangle.