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In any ΔABC the following relationship holds :

$$n_a \leq R + \sqrt{R^2 - rr_a}$$

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Let O, N_a be the circumcenter Point, Nagel Point in ΔABC, and let $\{D\} = (AN_a) \cap (BC)$.

By Stewart′s theorem in ΔBOC, we have

$$OD^2 = \frac{DB.OC^2 + DC.OB^2}{BC} - DB.DC = \frac{(s-c)R^2 + (s-b)R^2}{a} - (s-c)(s-b) = R^2 - rr_a.$$

Now, in ΔAOD, we have

$$AD \leq AO + OD \Leftrightarrow n_a \leq R + \sqrt{R^2 - rr_a},$$

as desired. Equality holds iff $b = c$.