ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC, the $following$ $relationship$ $holds$:

$$\frac{n_a + m_a}{\sqrt{n_a^2 + m_a w_a}} + \frac{n_b + m_b}{\sqrt{n_b^2 + m_b w_b}} + \frac{n_c + m_c}{\sqrt{n_c^2 + m_c w_c}} \geq 3\sqrt{2}$$

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$$\frac{n_a + m_a}{\sqrt{n_a^2 + m_a w_a}} \overset{?}{\geq} \sqrt{2} \Leftrightarrow n_a^2 + m_a^2 + 2n_a m_a \underbrace{\overset{?}{\geq}}_{(1)} 2n_a^2 + 2m_a w_a \textit{ and } \because 2m_a w_a$$

$\overset{\text{AM-GM}}{\leq}$ $m_a^2 + w_a^2$ $\therefore$ in order to prove (1), it suffices to prove : $m_a^2 + 2n_a m_a \overset{?}{\geq}$ $n_a^2 + m_a^2 + w_a^2 \Leftrightarrow 2n_a m_a - n_a^2 \overset{?}{\geq} w_a^2 \to$ true $\because 2n_a m_a - n_a^2 \overset{\text{Bogdan Fustei}}{\geq} s(s-a)$

$$\overset{\text{AM-GM}}{\geq} s(s-a).\frac{4bc}{(b+c)^2} = w_a^2$$

$\left(\because 2m_a \geq n_a + \dfrac{r_b r_c}{n_a}\right.$; Reference : Inequ$ali$ty in Tri$angle$ by Bogdan Fustei $-$ 42; published at www.ssmrmh.ro$\left.\right)$

$$\therefore \frac{n_a + m_a}{\sqrt{n_a^2 + m_a w_a}} \geq \sqrt{2} \textit{ and analogs } \therefore \frac{n_a + m_a}{\sqrt{n_a^2 + m_a w_a}} + \frac{n_b + m_b}{\sqrt{n_b^2 + m_b w_b}} + \frac{n_c + m_c}{\sqrt{n_c^2 + m_c w_c}}$$

$\geq 3\sqrt{2}\ \forall$ ABC, with equality iff Δ ABC is equil$ateral$ (QED)