ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC following relationship ho$l$ds :

$$\left(\frac{R}{2}-\frac{s}{3\sqrt{3}}+r\right)\left(\frac{2}{R}-\frac{3\sqrt{3}}{s}+\frac{1}{r}\right)\ge 1$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Soumava Chakraborty-Kolkata-India

$$\left(\frac{R}{2}-\frac{s}{3\sqrt{3}}+r\right)\left(\frac{2}{R}-\frac{3\sqrt{3}}{s}+\frac{1}{r}\right)=\frac{(3\sqrt{3}R-2s+6\sqrt{3}r)\left(r(2s-3\sqrt{3}R)+Rs\right)}{6\sqrt{3}Rrs}=$$

$$=\frac{-r(3\sqrt{3}R-2s)^2+(3\sqrt{3}R-2s)(Rs-6\sqrt{3}r^2)+6\sqrt{3}Rrs}{6\sqrt{3}Rrs}=$$

$$=1+\frac{(3\sqrt{3}R-2s)(Rs-6\sqrt{3}r^2-3\sqrt{3}Rr+2sr)}{6\sqrt{3}Rrs}=$$

$$=1+\frac{(3\sqrt{3}R-2s)\left((R+2r)s-3\sqrt{3}r(R+2r)\right)}{6\sqrt{3}Rrs}=$$

$$=1+\frac{(3\sqrt{3}R-2s)\left((R+2r)(s-3\sqrt{3}r)\right)}{6\sqrt{3}Rrs}\ge 1$$

$$\because (3\sqrt{3}R-2s),(s-3\sqrt{3}r)\overset{\text{Mitrinovic}}{\ge}0\ \therefore \left(\frac{R}{2}-\frac{s}{3\sqrt{3}}+r\right)\left(\frac{2}{R}-\frac{3\sqrt{3}}{s}+\frac{1}{r}\right)\ge 1\ \forall\ ABC,$$

$'' = ''$ iff Δ ABC is equi$lateral$ (QED)