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In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{|b-c|}{\sqrt{m_a}}\geq\sum_{cyc}\sqrt{\frac{a\left(n_a+p_a\sqrt{\frac{w_a}{g_a}}-2g_a\right)}{b+c}}$$

Proposed by Bogdan Fuștei-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

We have the following inequalities (see [1, pp. 1] and [2, pp. 2])

$$p_a\sqrt{\frac{w_a}{g_a}}\leq n_a \text{ and } n_a+g_a\geq 2m_a.$$

Then

$$\frac{a\left(n_a+p_a\sqrt{\frac{w_a}{g_a}}-2g_a\right)}{b+c}\leq\frac{2a(n_a-g_a)}{b+c}=\frac{2a({n_a}^2-{g_a}^2)}{(b+c)(n_a+g_a)}$$

$$\leq\frac{a\left[s\left(s-a+\frac{(b-c)^2}{a}\right)-(s-a)\left(s-\frac{(b-c)^2}{a}\right)\right]}{(b+c)m_a}=\frac{(b-c)^2}{m_a},$$

$$\Rightarrow\frac{|b-c|}{\sqrt{m_a}}\geq\sqrt{\frac{a\left(n_a+p_a\sqrt{\frac{w_a}{g_a}}-2g_a\right)}{b+c}}\quad\text{(and analogs)}$$

Adding this inequality with the similar ones yields the desired result.

Equality holds iff ΔABC is equilateral.

[1]. Bogdan Fuștei – CONNECTIONS BETWEEN FAMOUS CEVIANS-www.ssmrmh.ro

[2]. Bogdan Fuștei – ABOUT NAGEL AND GERGONNE CEVIANS (III)-www.ssmrmh.ro