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If $\mathrm{I} \to$ incenter in *any* Δ ABC then the following relationship holds :

$$\sum_{cyc}\left(AI\sqrt{\frac{n_a}{h_a}}\right) < \sum_{cyc}\sqrt{2R(n_a - |b-c|)}$$

Proposed by Bogdan Fuștei-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$AI^2.\frac{n_a}{h_a} = \frac{bc - 4Rr}{bc}.2Rn_a \overset{?}{<} 2R(n_a - |b-c|)$$

$$\Leftrightarrow -\frac{4Rr}{4Rrs}.2R.an_a \overset{?}{<} -2R|b-c| \Leftrightarrow |b-c| \overset{?}{<} \frac{an_a}{s}$$

$$\overset{\text{Bogdan Fustei}}{\Leftrightarrow} \quad |b-c| \overset{?}{<} \sqrt{4r^2 + (b-c)^2} \Leftrightarrow (b-c)^2 \overset{?}{<} 4r^2 + (b-c)^2 \to \text{true}$$

$$\therefore AI\sqrt{\frac{n_a}{h_a}} < \sqrt{2R(n_a - |b-c|)} \text{ and analogs}$$

$$\Rightarrow \sum_{cyc}\left(AI\sqrt{\frac{n_a}{h_a}}\right) < \sum_{cyc}\sqrt{2R(n_a - |b-c|)} \ \forall \Delta ABC \ (QED)$$