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In ΔABC the following relationship holds:

$$\frac{n_a(r_b-r_c)}{n_a-\sqrt{4r^2+(b-c)^2}}=\frac{s(b-c)}{r}$$

Proposed by Bogdan Fuștei-Romania

Solution by Tapas Das-India

$$\frac{n_a^2}{h_a^2}=1+\frac{(b-c)^2}{4r^2}\ or\ \frac{n_a}{h_a}.2r=\sqrt{4r^2+(b-c)^2}\ \ (1)$$

Reference:

ABOUT NAGEL AND GERGONNE'S CEVIANS by Bogdan Fuștei (www.ssmrmh.ro)

$$\frac{2r}{h_a}=\frac{2r}{\left(\frac{2rs}{a}\right)}=\frac{a}{s}\ (2)$$

$$r_b-r_c=\frac{F}{s-b}-\frac{F}{s-c}=\frac{F}{(s-b)(s-c)}(s-c-s+b)=\frac{F(b-c)}{(s-b)(s-c)}\ (3)$$

$$\frac{n_a(r_b-r_c)}{n_a-\sqrt{4r^2+(b-c)^2}}\overset{(1)}{=}\frac{n_a(r_b-r_c)}{n_a-\frac{n_a}{h_a}.2r}\overset{(2)}{=}\frac{(r_b-r_c)}{1-\frac{a}{s}}\overset{(3)}{=}$$

$$=\frac{F(b-c)}{(s-b)(s-c)}\cdot\frac{s}{s-a}=\frac{rs^2(b-c)}{sr^2}=\frac{s(b-c)}{r}$$