# ROMANIAN MATHEMATICAL MAGAZINE

$$\textit{If } a,b,c>0 \textit{ then prove:}$$

$$\sum_{cyc}\sqrt{\frac{a^4+b^4}{a}}\left(\sqrt{\frac{b+c}{b}}+\sqrt{\frac{c+a}{c}}\right)\ge 12\sqrt{abc}$$

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$$\frac{a^4+b^4}{a}\ge\frac{\frac{(a^2+b^2)^2}{2}}{a}=\frac{(a^2+b^2)^2}{2a}\ge\frac{4a^2b^2}{2a}\to\sqrt{\frac{a^4+b^4}{a}}\ge\frac{2ab}{\sqrt{2a}}$$

$$\textit{Analogously}:\ \sqrt{\frac{c^4+b^4}{b}}\ge\frac{2bc}{\sqrt{2b}},\qquad \sqrt{\frac{a^4+c^4}{c}}\ge\frac{2ca}{\sqrt{2c}}$$

$$LHS\ge\frac{2ab}{\sqrt{2a}}\left(\sqrt{\frac{b+c}{b}}+\sqrt{\frac{c+a}{c}}\right)+\frac{2bc}{\sqrt{2b}}\left(\sqrt{\frac{a+c}{c}}+\sqrt{\frac{b+a}{b}}\right)+$$

$$+\frac{2ca}{\sqrt{2c}}\left(\sqrt{\frac{b+a}{a}}+\sqrt{\frac{b+a}{b}}\right)\overset{A-G}{\ge}\frac{2ab}{\sqrt{2a}}.2\left(\sqrt{\frac{b+c}{b}.\frac{c+a}{c}}\right)^{\frac{1}{2}}+$$

$$+\frac{2bc}{\sqrt{2b}}.2\left(\sqrt{\frac{a+c}{c}.\frac{b+a}{a}}\right)^{\frac{1}{2}}+\frac{2ca}{\sqrt{2c}}.2\left(\sqrt{\frac{b+a}{a}.\frac{c+b}{b}}\right)^{\frac{1}{2}}\overset{A-G}{\ge}$$

$$\ge\frac{4ab}{\sqrt{2a}}.\left(\sqrt{\frac{4\sqrt{bc}}{b}.\frac{\sqrt{ca}}{c}}\right)^{\frac{1}{2}}+\frac{4ac}{\sqrt{2c}}.\left(\sqrt{\frac{4\sqrt{ba}}{a}.\frac{\sqrt{cb}}{b}}\right)^{\frac{1}{2}}+\frac{4bc}{\sqrt{2b}}.\left(\sqrt{\frac{4\sqrt{ca}}{c}.\frac{\sqrt{ab}}{a}}\right)^{\frac{1}{2}}\overset{A-G}{\ge}$$

$$\ge 3\left(\frac{4ab}{\sqrt{2a}}.\frac{4ac}{\sqrt{2c}}.\frac{4bc}{\sqrt{2b}}.\left(\sqrt{\frac{64a^2b^2c^2}{a^2b^2c^2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}=12\sqrt{abc}$$

$$\textit{Equality holds for}:\ a=b=c$$