ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\sum_{cyc}\sqrt{\frac{a^4+b^4}{a^2+b^2}}\left(\sqrt{\frac{b+c}{b}}+\sqrt{\frac{c+a}{c}}\right)\ge 6\sqrt{2}\sqrt[3]{abc}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\frac{a^4+b^4}{a^2+b^2}\overset{Chebyshev}{\ge}\frac{\frac{1}{2}(a^2+b^2)^2}{a^2+b^2}=\frac{1}{2}(a^2+b^2)\overset{CBS}{\ge}\frac{1}{4}(a+b)^2\ \ (1)$$

$$\sum_{cyc}\sqrt{\frac{a^4+b^4}{a^2+b^2}}\left(\sqrt{\frac{b+c}{b}}+\sqrt{\frac{c+a}{c}}\right)\overset{(1)}{\ge}\sum\frac{a+b}{2}\left(\sqrt{\frac{b+c}{b}}+\sqrt{\frac{c+a}{c}}\right)\ge$$

$$\overset{AM-GM}{\ge}\sum\frac{(a+b)\sqrt[4]{(b+c)(c+a)}}{\sqrt{bc}}\overset{AM-GM}{\ge}3\sqrt[3]{\frac{((a+b)(b+c)(c+a))^{\frac{3}{2}}}{\sqrt{abc}}}\ge$$

$$\overset{Cesaro}{\ge}3\sqrt[3]{\frac{(8abc)^{\frac{3}{2}}}{\sqrt{abc}}}=3\sqrt[3]{2^{\frac{9}{2}}abc}=3\times2\sqrt{2}\sqrt[3]{abc}=6\sqrt{2}\sqrt[3]{abc}$$

Equality holds for a= b=c.