ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ then:

$$\frac{\sqrt{a+b}}{a^2+b^2}\left(\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}\right)+\frac{\sqrt{b+c}}{b^2+c^2}\left(\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{a}}\right)+\frac{\sqrt{c+a}}{c^2+a^2}\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)\ge\frac{9\sqrt{2}}{a^2+b^2+c^2}$$

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Solution by Tapas Das-India

$$\frac{\sqrt{a+b}}{a^2+b^2}\left(\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}\right)=\frac{\sqrt{a+b}}{a^2+b^2}\left(\frac{1^{\frac{3}{2}}}{\sqrt{b}}+\frac{1^{\frac{3}{2}}}{\sqrt{c}}\right)\overset{Radon}{\ge}\frac{\sqrt{a+b}}{a^2+b^2}\frac{2\sqrt{2}}{\sqrt{b+c}}=\frac{2\sqrt{2}}{a^2+b^2}\sqrt{\frac{a+b}{b+c}}\ (1)$$

$$\frac{\sqrt{a+b}}{a^2+b^2}\left(\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}\right)+\frac{\sqrt{b+c}}{b^2+c^2}\left(\frac{1}{\sqrt{c}}+\frac{1}{\sqrt{a}}\right)+\frac{\sqrt{c+a}}{c^2+a^2}\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)=$$

$$=\sum\frac{\sqrt{a+b}}{a^2+b^2}\left(\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{c}}\right)\overset{(1)}{\ge}$$

$$\ge\sum\left(\frac{2\sqrt{2}}{a^2+b^2}\sqrt{\frac{a+b}{b+c}}\right)\overset{AM-GM}{\ge}6\sqrt{2}\sqrt[3]{\frac{1}{(a^2+b^2)(b^2+c^2)(c^2+a^2)}}\ge$$

$$\overset{AM-GM}{\ge}6\sqrt{2}\frac{1}{\left(\frac{2(a^2+b^2+c^2)}{3}\right)}=\frac{9\sqrt{2}}{a^2+b^2+c^2}$$

Equality holds for $a=b=c$.