

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$ and $x^2 + y^2 + z^2 \leq 12$ then:

$$\sqrt{3x^2 + 12xy + y^2} + \sqrt{3y^2 + 12yz + z^2} + \sqrt{3z^2 + 12zx + x^2} \leq 24$$

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Solution by Tapas Das-India

$$\begin{aligned} & \sqrt{3x^2 + 12xy + y^2} + \sqrt{3y^2 + 12yz + z^2} + \sqrt{3z^2 + 12zx + x^2} \leq \\ & \stackrel{CBS}{\leq} \sqrt{3 \cdot \sum_{cyc} (3x^2 + 12xy + y^2)} = \sqrt{3 \left(3 \sum x^2 + 12 \sum xy + \sum y^2 \right)} \leq \\ & \leq \sqrt{3 \left(3 \sum x^2 + 12 \sum x^2 + \sum y^2 \right)} \stackrel{x^2+y^2+z^2 \leq 12}{\leq} \\ & \leq \sqrt{3(3 \times 12 + 12 \times 12 + 12)} = \sqrt{576} = 24 \end{aligned}$$

Equality holds for $x=y=z=2$.