ROMANIAN MATHEMATICAL MAGAZINE

If $x, y > 0, x + y = 2$ then:

$$\sqrt{10x^2 + 4xy + 2y^2} + \sqrt{10y^2 + 4xy + 2x^2} \geq 8$$

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$\textit{Lemma}$:

$$\forall\ a, b > 0, \sqrt{10a^2 + 4ab + 2b^2} \geq 3a + b$$

$\textit{Proof}$:

$$\textit{We need to show}:$$
$$\sqrt{10a^2 + 4ab + 2b^2} \geq 3a + b$$
$$10a^2 + 4ab + 2b^2 \overset{sqaring}{\geq} (3a + b)^2$$
$$10a^2 + 4ab + 2b^2 \geq 9a^2 + 6ab + b^2$$
$$a^2 - 2ab + b^2 \geq 0\ or, (a - b)^2 \geq 0\ true$$

$$\sqrt{10x^2 + 4xy + 2y^2} + \sqrt{10y^2 + 4xy + 2x^2} \overset{Lemma}{\geq}$$
$$\geq (\,3x + y) + (3y + x) = 4(x + y) \overset{x+y=2}{=} 8$$

Equality holds for x=y=2.