

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z > 0$, $x + y + z \leq 1$ then:

$$2(x + y + z) + 3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \geq 29$$

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Solution by Tapas Das-India

We need to show :

$$2(x + y + z) + 3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) \geq 29$$

$$2(x + y + z) + 3\frac{(1 + 1 + 1)^2}{x + y + z} \geq 29$$

$$2t + \frac{27}{t} \geq 29 \quad \text{where } x+y+z=t$$

$2t^2 - 29t + 27 \geq 0$ or $(2t - 27)(t - 1) \geq 0$ true
since $t = x + y + z \leq 1$ (given) so, $(t - 1) \leq 0$ and
 $(2t - 27) < 0$ consequently: $(2t - 27)(t - 1) \geq 0$

Equality holds for $x = y = z = \frac{1}{3}$