ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0, 17a+14b+23c=54$ then:

$$\sqrt{4a^2+2ab+3b^2}+2\sqrt{4b^2+2ab+3c^2}+3\sqrt{4c^2+2ab+3a^2}\ge 18$$

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$\boldsymbol{Lemma}$:

$$\forall\, x,y>0\ \sqrt{4x^2+2xy+3z^2}\ge\frac{5x+4y}{3}$$

$\boldsymbol{Proof}$:

$$\sqrt{4x^2+2xy+3z^2}\ge\frac{5x+4y}{3}$$

$$or\ 9(4x^2+2xy+3y^2)\overset{squaring}{\ge}25x^2+16y^2+40xy$$

$$or\ 11x^2-22xy+11y^2\ge 0\ or\ 11(x-y)^2\ge 0\ \ true$$

$$\sqrt{4a^2+2ab+3b^2}+2\sqrt{4b^2+2ab+3c^2}+3\sqrt{4c^2+2ab+3a^2}\ge$$

$$\overset{Lemma}{\ge}\frac{5a+4b}{3}+2.\frac{5b+4c}{3}+3.\frac{5c+4a}{3}=\frac{1}{3}(17a+14b+23c)=$$

$$\overset{17a+14b+23c=54}{=}\frac{1}{3}\times 54=18$$

Equality holds for a=b=c=1.