ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c > 0,\ a + b + c = 3$ then:

$$\frac{a^2 + b^2}{c^2 + 1} + \frac{b^2 + c^2}{a^2 + 1} + \frac{c^2 + a^2}{b^2 + 1} \geq 3$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$We\ know\ that\ 3(a^2 + b^2 + c^2) \geq (a + b + c)^2 = 9\ (as\ a + b + c = 3\ )\ or$$
$$(a^2 + b^2 + c^2) \geq 3\ (1)$$

$$We\ need\ to\ show,:$$
$$\frac{a^2 + b^2}{c^2 + 1} + \frac{b^2 + c^2}{a^2 + 1} + \frac{c^2 + a^2}{b^2 + 1} \geq 3$$
$$or \sum \frac{a^2 + b^2}{c^2 + 1} \geq 3\ or \sum \left(\frac{a^2 + b^2}{c^2 + 1} + 1\right) \geq 6$$
$$or \sum \frac{(a^2 + b^2 + c^2) + 1}{c^2 + 1} \geq 6\ or\ (a^2 + b^2 + c^2) \sum \frac{1}{c^2 + 1} + \sum \frac{1}{c^2 + 1} \geq 6$$
$$or\ (a^2 + b^2 + c^2) \frac{(1 + 1 + 1)^2}{a^2 + b^2 + c^2 + 3} + \frac{(1 + 1 + 1)^2}{a^2 + b^2 + c^2 + 3} \overset{CBS}{\geq} 6$$
$$or \frac{9x + 9}{x + 3} \overset{(a^2+b^2+c^2)=x}{\geq} 6\ or\ 9x + 9 \geq 6x + 18\ or\ x \geq 3$$
$$or\ (a^2 + b^2 + c^2) \geq 3\ true\ by\ (1)$$

Equality holds for a=b=c=1.