ROMANIAN MATHEMATICAL MAGAZINE

If $a, \mathrm{b}, \mathrm{c} > 0, a + \mathrm{b} + \mathrm{c} = \frac{3}{2}$ and $\lambda \geq 0$ then :

$$\sum_{\mathrm{cyc}} \frac{1 + \lambda \mathrm{b}}{1 + 4a^2} \geq \frac{3}{4}(\lambda + 2)$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\frac{1}{1 + 4a^2} \overset{?}{\geq} 1 - a \Leftrightarrow a(2a - 1)^2 \overset{?}{\geq} 0 \to \text{true}$$

$$\therefore \frac{1 + \lambda \mathrm{b}}{1 + 4a^2} \geq (1 + \lambda \mathrm{b})(1 - a) \text{ and analogs}$$

$$\therefore \sum_{\mathrm{cyc}} \frac{1 + \lambda \mathrm{b}}{1 + 4a^2} \geq \sum_{\mathrm{cyc}} (1 - a + \lambda \mathrm{b} - \lambda a \mathrm{b}) \overset{a+\mathrm{b}+\mathrm{c} = \frac{3}{2}}{=} 3 - \frac{3}{2} + \lambda . \frac{3}{2} - \lambda \sum_{\mathrm{cyc}} a\mathrm{b} \geq$$

$$\frac{3}{2} + \lambda . \frac{3}{2} - \frac{\lambda}{3}\left(\sum_{\mathrm{cyc}} a\right)^2 \ (\because \lambda \geq 0) = \frac{3}{2} + \lambda . \frac{3}{2} - \frac{\lambda}{3} . \frac{9}{4} = \frac{3}{4}(\lambda + 2)$$

$$\therefore \sum_{\mathrm{cyc}} \frac{1 + \lambda \mathrm{b}}{1 + 4a^2} \geq \frac{3}{4}(\lambda + 2) \ \forall \ a, \mathrm{b}, \mathrm{c} > 0 \,\Big|\, a + \mathrm{b} + \mathrm{c} = \frac{3}{2} \text{ and } \lambda \geq 0,$$

$$'' ='' \text{ iff } a = \mathrm{b} = \mathrm{c} = \frac{1}{2} \ (\text{QED})$$