ROMANIAN MATHEMATICAL MAGAZINE

If $a,b,c>0$ and $a+b+c=3,\lambda,n\geq 0$ then:

$$\sum\frac{a^3+\ \lambda b^3}{a+nb}\geq\frac{3(\lambda+1)}{n+1}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum\frac{a^3+\ \lambda b^3}{a+nb}=\sum\frac{a^3}{a+nb}+\lambda\sum\frac{b^3}{a+nb}\overset{Holder}{\geq}$$

$$\geq\frac{1}{3}\frac{(a+b+c)^3}{(a+b+c)+n(a+b+c)}+\frac{\lambda}{3}\frac{(a+b+c)^3}{(a+b+c)+n(a+b+c)}=$$

$$=\frac{1}{3}\frac{(a+b+c)^2}{(n+1)}+\frac{\lambda}{3}\frac{(a+b+c)^2}{(n+1)}\overset{a+b+c=3}{=}\frac{3(\lambda+1)}{n+1}$$

$$Equality\ holds\ for\ a=b=c=1.$$