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If $x, y, z > 0$, $x + y + z = 3$, $\lambda > 0$ then:

$$\sum \frac{x}{\sqrt{\lambda(y+z) + \lambda}} \geq \sqrt{\frac{3}{\lambda}}$$

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Solution by Tapas Das-India

$$\begin{aligned} \sum \frac{x}{\sqrt{\lambda(y+z) + \lambda}} &= \frac{1}{\sqrt{\lambda}} \sum \frac{x}{\sqrt{(y+z) + 1}} = \\ &= \frac{1}{\sqrt{\lambda}} \sum \frac{x^{\frac{3}{2}}}{\sqrt{(yx + zx) + x}} \stackrel{\text{Radon}}{\geq} \frac{1}{\sqrt{\lambda}} \frac{(x+y+z)^{\frac{3}{2}}}{\sqrt{2(xy + yz + zx) + x + y + z}} \geq \\ &\geq \frac{1}{\sqrt{\lambda}} \frac{(x+y+z)^{\frac{3}{2}}}{\sqrt{\frac{2(x+y+z)^2}{3} + x + y + z}} \stackrel{x+y+z=3}{=} \frac{1}{\sqrt{\lambda}} \frac{3\sqrt{3}}{3} = \sqrt{\frac{3}{\lambda}} \end{aligned}$$

Equality holds for $x=y=z=1$.