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**If $a,b,c>0,\ n\in N$ then:**

$$\sum a^{n+2}+\sum\frac{a^n}{(b+c)^2}\ge 3\left(\frac{\sum a}{3}\right)^n$$

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*Solution by Tapas Das-India*

$$\sum a^{n+2}=\sum\frac{a^{n+2}}{1^{n+1}}\overset{Radon}{\ge}\frac{1}{3^{n+1}}\left(\sum a\right)^{n+2}\ (1)$$

$$\sum\frac{a^n}{(b+c)^2}\overset{Chebyshev}{\ge}\frac{1}{3}\left(\sum a^n\right)\left(\sum\frac{1}{(b+c)^2}\right)=$$
$$=\frac{1}{3}\left(\sum\frac{a^n}{1^{n-1}}\right)\left(\sum\frac{1^3}{(b+c)^2}\right)\overset{Radon}{\ge}\frac{1}{3}\cdot\frac{(\sum a)^n}{3^{n-1}}\cdot\frac{27}{4(\sum a)^2}\ (2)$$

$$\sum a^{n+2}+\sum\frac{a^n}{(b+c)^2}\overset{(1)\&(2)}{\ge}$$
$$\ge\frac{1}{3^{n+1}}\left(\sum a\right)^{n+2}+\frac{1}{3}\cdot\frac{(\sum a)^n}{3^{n-1}}\cdot\frac{27}{4(\sum a)^2}\overset{AM-GM}{\ge}$$
$$\ge 2\sqrt{\frac{1}{3^{n+1}}\left(\sum a\right)^{n+2}\cdot\frac{1}{3}\cdot\frac{(\sum a)^n}{3^{n-1}}\cdot\frac{27}{4(\sum a)^2}}=2\left(\sum a\right)^n\cdot\frac{3}{2}\cdot\frac{1}{3^n}=3\left(\frac{\sum a}{3}\right)^n$$

**Equality holds for a=b=c=1.**