ROMANIAN MATHEMATICAL MAGAZINE

If $a, b > 0, a + b \geq 2, 0 < \lambda \leq 1$ then:

$$\frac{4(a^4+b^4)}{a^2+b^2} - \frac{1}{2}(a+b)^2 + \lambda\left(\frac{1}{a^2}+\frac{1}{b^2}\right) \geq 2(\lambda+1)$$

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$$Let\ t = (a+b)^2 \overset{a+b\geq 2}{\geq} 2^2 = 4\ (1)$$

$$0 \leq \lambda \leq 1\ then\ 0 < 4\lambda \leq 4\ then\ (t-4\lambda) > 0\ and\ (t-4) \geq 0\ as\ \ t \geq 4\ (2)$$

$$a^4+b^4 = \frac{(a^2)^2}{1} + \frac{(b^2)^2}{1} \overset{Radon}{\geq} \frac{(a^2+b^2)^2}{2}\ \ then$$

$$\frac{4(a^4+b^4)}{a^2+b^2} \geq \frac{4.\frac{(a^2+b^2)^2}{2}}{(a^2+b^2)} = 2(a^2+b^2) = 2\left(\frac{a^2}{1}+\frac{b^2}{1}\right) \overset{Radon}{\geq} (a+b)^2\ (3)$$

$$\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = \left(\frac{1^3}{a^2}+\frac{1^3}{b^2}\right) \overset{Radon}{\geq} \frac{8}{(a+b)^3}\ (4)$$

$$We\ need\ to\ show:$$

$$\frac{4(a^4+b^4)}{a^2+b^2} - \frac{1}{2}(a+b)^2 + \lambda\left(\frac{1}{a^2}+\frac{1}{b^2}\right) \geq 2(\lambda+1)$$

$$(a+b)^2 - \frac{1}{2}(a+b)^2 + \lambda\frac{8}{(a+b)^3} \overset{(4)\&(3)}{\geq} 2(\lambda+1)$$

$$\frac{t}{2} + \frac{8\lambda}{t} \overset{t=(a+b)^2}{\geq} 2(\lambda+1)$$

$$t^2 - 4t\lambda - 4 + 16\lambda \geq 0$$

$$(t-4\lambda)\ (t-4) \geq 0\ true\ by\ (2)$$

Equality holds for a=b=1.