

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\begin{cases} x(y+z) = 35 \\ y(x+z) = 35 \\ z(x+y) = 27 \end{cases}$$

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$$x(y+z) = 35 \quad (1), \quad y(x+z) = 35 \quad (2), \quad z(x+y) = 27 \quad (3)$$

From(1)&(2) we get $x(y+z) = y(x+z)$ *or* $z(x-y) = 0 \Rightarrow z = 0$ *or* $x = y$

If $z = 0$ *then from (3)we get* $0 = 27$, *not possible so* $z \neq 0$

If $x = y = m$ (say) *then from(3)we have* $z \cdot (2m) = 27$ *or,* $z = \frac{27}{2m}$

From(1): $m(m+z) = 35$ *or,* $m\left(m + \frac{27}{2m}\right) = 35$ *or* $m^2 = \frac{43}{2}$ *or*

$$m = \pm \sqrt{\frac{43}{2}}, z = \frac{27}{2m} = \pm \frac{27}{\sqrt{86}}$$

So required solutions: $x = \sqrt{\frac{43}{2}}, y = \sqrt{\frac{43}{2}}, z = \frac{27}{\sqrt{86}}$ *and*

$$x = -\sqrt{\frac{43}{2}}, y = -\sqrt{\frac{43}{2}}, z = -\frac{27}{\sqrt{86}}$$