ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\begin{cases} x+\frac{1}{z}=3 \\ y+\frac{1}{x}=2 \\ xyz=1 \end{cases}$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$x+\frac{1}{z}=3\ then\ z=\frac{1}{3-x}\ ,y+\frac{1}{x}=2\ then\ x=\frac{1}{2-y}$$

$$xyz=1\ or\ \frac{y}{(3-x)(2-y)}=1\ or\ \frac{y}{\left(3-\frac{1}{2-y}\right)(2-y)}=1$$

$$\frac{y}{5-3y}=1\ or\ y=\frac{5}{4}$$

$$x=\frac{1}{2-y}=\frac{1}{2-\frac{5}{4}}=\frac{4}{3}and\ z=\frac{1}{xy}=\frac{1}{\frac{4}{3}.\frac{5}{4}}=\frac{3}{5}$$