ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers and $m \in R$ fixed:

$$\begin{cases} x+y-z=m-2 \\ x^2+y^2-z^2=m^2-4m-8 \\ x^3+y^3-z^3=m^3-6m^2-24m-26 \end{cases}$$

*Proposed by Marin Chirciu-Romania*

*Solution by Tapas Das-India*

$$Let\ a=x+y, p=xy$$

$$From\ 1st\ equation: x+y-z=m-2\ or, z=a-(m-2)\ (1)$$

$$x^2+y^2=(x+y)^2-2xy=a^2-2p$$

$$From\ 2nd\ equation: x^2+y^2-z^2=m^2-4m-8$$

$$a^2-2p-(a-(m-2))^2=m^2-4m-8$$

$$-2p+2a(m-2)-(m-2)^2=m^2-4m-8$$

$$-2p=m^2-4m-8+(m-2)^2-2a(m-2)$$

$$-2p=2m^2-8m-4-2a(m-2) or, p=a(m-2)-m^2+4m+2\ (2)$$

$$x^3+y^3=(x+y)^3-3xy(x+y)=a^3-3ap$$

$$From\ 3rd\ equation: x^3+y^3-z^3=m^3-6m^2-24m-26$$

$$a^3-3ap-\left(a-(m-2)\right)^3\overset{(1)}{=}m^3-6m^2-24m-26$$

$$a^3-3ap-a^3+(m-2)^3+3a^2(m-2)-3a(m-2)^2=m^3-6m^2-24m-26$$

$$3a(-p+a(m-2)-(m-2)^2)=m^3-6m^2-24m-26-(m-2)^3$$

$$3a(-a(m-2)+m^2-4m-2+a(m-2)-(m-2)^2)\overset{(2)}{=}-36m-18$$

$$3a\times(-6)=-36m-18\ or\ a=2m+1$$

$$From\ (1):$$

$$z=a-(m-2)=2m+1-(m-2)=m+3\ (3)$$

$$From\ (2):$$

$$p=a(m-2)-m^2+4m+2=(2m+1)(m-2)-m^2+4m+2=m^2+m$$

$$now\ we\ have\ a=x+y=2m+1, and\ p=xy=m^2+m\ , so\ x,y\ are\ roots\ of$$

$$t^2-(2m+1)t+(m^2+m)=0\ or$$

$$t=\frac{2m+1\pm\sqrt{(2m+1)^2-4.1.(m^2+m)}}{2}=\frac{2m+1\pm 1}{2}=m, m+1$$

$$so\ (x,y)=(m,m+1) or, (m+1,m)$$

$$Required\ solution\ using\ (3): (x,y,z)=(m,m+1,m+3), (m+1,m,m+3)$$