ROMANIAN MATHEMATICAL MAGAZINE

If $\begin{cases} a+b+c=0 \\ ab+ac+bc=-3 \\ bac=2 \end{cases}$ then find $\dfrac{\sum a^2 \sum a^6}{\sum a^3 \sum a^5}$

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We know that:

$$\sum a^2 = \left(\sum a\right)^2 - 2\sum ab \Rightarrow \sum a^2 = 6$$

a,b and c are the roots of the equation:

$$x^3 - 3x - 2 = 0$$

$$\begin{cases} \sum a^3 - 3\sum a - 6 = 0 \\ \sum a^4 - 3\sum a^2 - 2\sum a = 0 \\ \sum a^5 - 3\sum a^3 - 2\sum a^2 = 0 \\ \sum a^6 - 3\sum a^4 - 2\sum a^3 = 0 \end{cases} \Rightarrow \begin{cases} \sum a^3 = 6 \\ \sum a^4 = 18 \\ \sum a^5 = 30 \\ \sum a^6 = 66 \end{cases} \Rightarrow \frac{\sum a^2 \sum a^6}{\sum a^3 \sum a^5} = 2,2$$