ROMANIAN MATHEMATICAL MAGAZINE

Find $x,y,z\in R^+$ such that:

$$\begin{cases}\sqrt{3}(x-y)\le 1+xy\\ \sqrt{3}(y-z)\le 1+yz\\ \sqrt{3}(1+xz)\le x-z\end{cases}$$

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Solution by Amin Hajiyev-Azerbaijan

$$\begin{cases}\frac{x-y}{1+xy}\le\frac{1}{\sqrt{3}}\\ \frac{y-z}{1+yz}\le\frac{1}{\sqrt{3}}\\ \frac{x-z}{1+xz}\ge\sqrt{3}\end{cases}\rightarrow \tan(a-b)=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}.\rightarrow\begin{cases}x=\tan(\alpha)\\ y=\tan(\beta)\\ z=\tan(\gamma)\end{cases}$$

$$x,y,z\in R^+\rightarrow\alpha,\beta,\gamma\in\left(0;\frac{\pi}{2}\right)$$

$$\begin{cases}\tan(\alpha-\beta)\le\frac{1}{\sqrt{3}}\\ \tan(\beta-\gamma)\le\frac{1}{\sqrt{3}}\\ \tan(\alpha-\gamma)\ge\sqrt{3}\end{cases}\rightarrow\begin{cases}\alpha-\beta\le\frac{\pi}{6}\\ \beta-\gamma\le\frac{\pi}{6}\\ \alpha-\gamma\ge\frac{\pi}{3}\end{cases}\rightarrow\begin{cases}\alpha-\beta+\beta-\gamma\le\frac{\pi}{3}\\ \alpha-\gamma\ge\frac{\pi}{3}\end{cases}\rightarrow\begin{cases}\alpha-\gamma\le\frac{\pi}{3}\\ \alpha-\gamma\ge\frac{\pi}{3}\end{cases}$$

$$\alpha-\gamma=\frac{\pi}{3},\qquad \alpha-\beta=\frac{\pi}{6},\qquad \beta-\gamma=\frac{\pi}{6}$$

$$1.\ \sqrt{3}x-\sqrt{3}y-xy=1\quad y=\frac{x\sqrt{3}-1}{x+\sqrt{3}}\quad y\in R^+\quad x>\frac{1}{\sqrt{3}}$$

$$2.\ x-z=\sqrt{3}(1+xz)\quad z=\frac{x-\sqrt{3}}{x\sqrt{3}+1}\quad z\in R^+\quad x>\sqrt{3}$$

$$x>\sqrt{3}$$

$$\textit{The solution set of the system: }x,y,z\in R^+\rightarrow\left(x>\sqrt{3};\ \frac{x\sqrt{3}-1}{x+\sqrt{3}};\frac{x-\sqrt{3}}{x\sqrt{3}+1}\right)$$