ROMANIAN MATHEMATICAL MAGAZINE

Find $x, y, z \in Z$ such that:

$$2x^2 + 3y^2 = z^2 - 37$$
$$x - y + z = 3$$

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$$2x^2 + 3y^2 = z^2 - 37 \ or, 2x^2 + 3y^2 \overset{x-y+z=3}{=} (3 - x + y)^2 - 37$$
$$or, \quad 2x^2 + 3y^2 = x^2 + y^2 + 9 - 6x + 6y - 2xy - 37$$
$$or, x^2 + (2y + 6)x + 2y^2 - 6y + 28 = 0 \ (1)$$

$$Discriminant = D = (2y + 6)^2 - 4(2y^2 - 6y + 28) = 4(17 - (y - 6)^2)$$
$$for\ integer\ solution\ (17 - (y - 6)^2) \geq 0\ and\ must\ be\ perfect\ square.$$
$$for\ this\ (y - 6)^2 = 1, 16\ so, y = 2, 5, 7, 10\ , z = 3 - x + y$$

$$for\ y = 7: \ from(1)\ x^2 + 20x + 84 = 0\ or, (x + 6)(x + 14) = 0\ or, x = -6, -14$$
$$and\ z = 3 - x + y = 3 - (-6) + 7 = 16\ and\ z = 3 - (-14) + 7 = 24$$
$$solution = (x, y, z) = \{(-6, 7, 16), (-14, 7, 24)\}$$

$$for\ y = 5: from(1)\ x^2 + 16x + 48 = 0\ or, (x + 4)(x + 12) = 0\ or, x = -4, -12$$
$$and\ z = 3 - x + y = 3 - (-4) + 5 = 12\ and\ z = 3 - (-12) + 5 = 20$$
$$solution = (x, y, z) = \{(-4, 5, 12), (-4, 5, 20)\}$$

$$for\ y = 2: \ from(1)\ x^2 + 10x + 24 = 0\ or, (x + 6)(x + 4) = 0\ or, x = -6, -4$$
$$and\ z = 3 - x + y = 3 - (-4) + 2 = 9\ and\ z = 3 - (-6) + 2 = 11$$
$$solution = (x, y, z) = \{(-6, 2, 9), (-4, 2, 11)\}$$

$$for\ y = 10\ from(1)\ x^2 + 26x + 168 = 0\ or, (x + 12)(x + 14) = 0\ or, x = -14, -12$$
$$and\ z = 3 - x + y = 3 - (-14) + 10 = 27\ and\ z = 3 - (-12) + 10 = 25$$
$$solution = (x, y, z) = \{(-14, 10, 10), (-12, 10, 25)\}$$