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Find the last digit of $A = 2024^{2025^{2026}} + 2026^{2027^{2028}} + 2028^{2029^{2030}}$

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Now we need to find the last digit of $2024^{2025^{2026}}$

last digit of 2024 is 4

now, 4^1 ends in 4, 4^2 ends in 6, 4^3 ends in 4

the pattern is 4, 6 which repeats every 2 powers (1)

We know that any positive integer power of an odd number odd 2025 is an odd then 2025^{2026} is an odd number, if exponent is odd, the last digit of $2024^{2025^{2026}}$ is 4 (using (1)) (2)

now we need to find the last digit of $2026^{2027^{2028}}$

clearly, $6^1 = 6, 6^2 = 36, 6^3 = 216, \dots$

so clearly last digit of $2026^{2027^{2028}}$ is 6 (3)

now we need to find the last digit of $2028^{2029^{2030}}$

for this we can focus on the last digit of 2028,

which is 8 and

$8^1 = 8, 8^2$ ends in 4, 8^3 ends in 2, 8^4 ends in 6

the pattern repeats every 4 power

now, $2029 \equiv 1 \pmod{4}, (2029)^{2030} \equiv 1^{2030} \pmod{4}, 2029^{2030} \equiv 1 \pmod{4}$

So, last digit of $2028^{2029^{2030}}$ is $8^1 = 8$ (4)

**Now using result (2), (3), (4) we get $4 + 6 + 8 = 18$
last digit of A is 8**