ROMANIAN MATHEMATICAL MAGAZINE

Let be $f, g, p: \mathbb{R} \to \mathbb{R}$, bijectifs functions such that:

$$p(x) = 13x - 15, f^{-1}(-1) = 1.5,$$

$$f\left(g^2(x) - 2g(x)\right) = \frac{3(x+1)}{3x+2}, x \in \mathbb{R} - \{1, -\frac{2}{3}\}$$

Find: $p\left(g^{-1}(1)\right)$.

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Solution by Shirvan Tahirov-Azerbaijan

$$f\left(g^2(x) - 2g(x)\right) = \frac{3(x+1)}{3x+2} = t$$

$$x = \frac{3t^{-1} + 3}{3t^{-1} + 2} \to \begin{cases} 3xt^{-1} - 3t^{-1} + 2x = 3 \\ 3xt^{-1} - 3t^{-1} = 3 - 2x \\ t^{-1}(3x - 3) = 3 - 2x \\ t^{-1} = \dfrac{3 - 2x}{3x - 3} \end{cases}$$

$$f^{-1}\left(g^2(x) - 2g(x)\right) = \frac{3 - 2x}{3x - 3}$$

$$p\left(g^{-1}(1)\right) \to \quad g^{-1}(1) = x \ , \qquad g(x) = 1$$

$$\frac{3 - 2x}{3x - 3} = 1.5 \to \begin{cases} 4.5x - 4.5 = 3 - 2x \\ x = \dfrac{15}{13} \end{cases}$$

$$g\left(\frac{15}{13}\right) = 1, \ \ g^{-1}(1) = \frac{15}{13}$$

$$p\left(g^{-1}(1)\right) = p\left(\frac{15}{13}\right) = 13.\frac{15}{13} - 15 = 0$$