ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$x^8+18x^7+32x^6-36x^5-146x^4-64x^3+128x^2+128x+32=0$$

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Solution by proposer

$$x^8+16x^7+2x^7+32x^6-32x^5-4x^5-16x^4-64x^4-64x^4-2x^4-32x^3-32x^3+128x^2+64x+64x+32=0\to(x^8+16x^7-32x^5-16x^4)+(2x^7+32x^6-64x^4-32x^3)+(-4x^5-64x^4+128x^2+64x)+(-2x^4-32x^3+64x+32)=0\to$$

$$\to x^4(x^4+16x^3-32x-16)+2x^3(x^4+16x^3-32x-16)-4x(x^4+16x^3-32x-16)-2(x^4+16x^3-32x-16)=0$$

$$(x^4+16x^3-32x-16)(x^4+2x^3-4x-2)=0$$

$$x^4+16x^3-32x-16=0\to\ 16(x^3-2x-1)=-x^4\to(x+1)(x^2-x-1)=-\frac{1}{16}x^4\frac{(x+1)}{x^2}\frac{(x^2-x-1)}{x^2}=-\frac{1}{16}$$

$$\left(\frac{x+1}{x^2}\right)\left(\frac{x^2-(x+1)}{x^2}\right)=-\frac{1}{16}$$

$$\frac{x+1}{x^2}=t\ ,\ t^2-t-\frac{1}{16}=0$$

$$x_{1,2}=\frac{1\pm\sqrt{3+\sqrt{5}}}{1+\frac{\sqrt{5}}{2}}=\frac{1\pm\frac{1+\sqrt{5}}{\sqrt{2}}}{1+\frac{\sqrt{5}}{2}}=\frac{2\pm(\sqrt{2}+\sqrt{10})}{2+\sqrt{5}}$$

$$x_{3,4}=\frac{1\pm\sqrt{3-\sqrt{5}}}{1-\frac{\sqrt{5}}{2}}=\frac{1\pm\frac{-1+\sqrt{5}}{\sqrt{2}}}{1-\frac{\sqrt{5}}{2}}=\frac{2\pm(\sqrt{10}-\sqrt{2})}{2-\sqrt{5}}$$

$$x^4+2x^3-4x-2=0$$

In the same manner:

$$x_{5,6}=\frac{1\pm\sqrt{3+2\sqrt{3}}}{1+\sqrt{3}},\qquad x_{7,8}=\frac{1\pm\sqrt{3-2\sqrt{3}}}{1-\sqrt{3}}$$

$$3-2\sqrt{3}<0,\qquad x_{7,8}=\frac{1\pm\sqrt{2\sqrt{3}-3}i}{1-\sqrt{3}}$$

Here $i^2=-1$