

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$x^5 - \sqrt[5]{x+30} = 30$$

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$$\text{Let } f(x) = x^5 - \sqrt[5]{x+30} - 30$$

at $x = 2, f(2) = 32 - 2 - 30 = 0$ so $x = 2$ is a solution of $f(x) = 0$

$$\text{now } f'(x) = 5x^4 - \frac{1}{5}(x+30)^{-\frac{4}{5}} > 0, \forall x \in \mathbb{R}$$

so $f(x)$ is strictly increasing

For this $f(x) = 0$ has at most one zero so required solution $x = 2$.