ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$2x^2-14x+30-5(x-4)\sqrt{x-1}=0$$

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$$2x^2-14x+30-5(x-4)\sqrt{x-1}=0$$

$$Clearly\ x-1\geq 0\ or\ x\geq 1$$

$$Let\ y=\sqrt{x-1}\Rightarrow x=y^2+1\quad (1)$$

$$Using\ (1) given\ equation\ can\ be\ written\ as:$$

$$2(y^2+1)^2-14(y^2+1)+30-5y(y^2-3)=0$$

$$2(y^4+2y^2+1)-(14y^2+14)+30-(5y^3-15y)=0$$

$$2y^4-5y^3-10y^2+15y+18=0$$

$$(y-2)(y+1)(y-3)(2y+3)=0$$

$$y-2=0\Rightarrow y=2\ then\ x=y^2+1=2^2+1=5$$

$$y+1=0\Rightarrow y=-1\ then:$$
$$x=y^2+1=(-1)^2+1=2\ ,not\ satisfied\ given\ equation$$

$$y-3=0\Rightarrow y=3\ then\ x=y^2+1=9^2+1=10$$
$$2y+3=0\Rightarrow y=-\frac{3}{2}\ then$$
$$x=\left(-\frac{3}{2}\right)^2+1=\frac{13}{4}=5, not\ satisfied\ given\ equation$$

$$So\ required\ solution\ x=5,10$$