Solve for reals:

$$\sqrt[3]{x - 2024} + \sqrt[3]{x - 2025} + \sqrt[3]{x - 2026} = 0$$

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Solution by Tapas Das-India

$$Let\ y = x - 2025$$

The given equation can be written as:

$$\sqrt[3]{y + 1} + \sqrt[3]{y} + \sqrt[3]{y - 1} = 0$$

Let $f(y) = \sqrt[3]{y + 1} + \sqrt[3]{y} + \sqrt[3]{y - 1}$ *clearly* $f(0) = 0$ *so* $y = 0$ *is a solution of*

$$f(y) = 0$$

$$f'(y) = \frac{1}{3}\left(\frac{1}{(y + 1)^{\frac{2}{3}}} + \frac{1}{y^{\frac{2}{3}}} + \frac{1}{(y - 1)^{\frac{2}{3}}}\right) > 0\ \forall\ y \in R - \{0, 1, -1\}$$

So $f(y)$ *is strictly increasing continuous function can have at most one zero ,*
so required solution $y = 0$ *or* $x - 2025 = 0$ *or* $x = 2025$