

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt[4]{1+x} + \sqrt[4]{1-x} = 2$$

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$$\text{Denote: } \begin{cases} a = \sqrt[4]{1+x} \\ b = \sqrt[4]{1-x} \end{cases}, \begin{cases} S = a + b \\ P = ab \end{cases}$$

$$\begin{cases} a^4 + b^4 = 2 \\ a + b = 2 \end{cases} \Rightarrow \begin{cases} (S^2 - 2P)^2 - 2P^2 = 2 \\ S = 2 \end{cases}$$

$$(4 - 2P)^2 - 2P^2 = 2 \Rightarrow 16 - 16P + 4P^2 - 2P^2 = 2$$

$$2P^2 - 16P + 14 = 0 \Rightarrow P^2 - 8P + 7 = 0$$

$$P = 7; S = 2 \Rightarrow z^2 - Sz + P = 0 \Rightarrow z^2 - 2z + 7 = 0; \Delta < 0$$

$$P = 1; S = 2 \Rightarrow a = b = 1 \Rightarrow 1 - x = 1 + x = 1 \Rightarrow x = 0$$