

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt[3]{(1+x)^2} + \sqrt[3]{(1-x)^2} - \sqrt[3]{1-x^2} = 1$$

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Denote:

$$\begin{cases} \sqrt[3]{1+x} = a \\ \sqrt[3]{1-x} = b \end{cases} \Rightarrow a^3 + b^3 = 1+x+1-x = 2$$

$$\sqrt[3]{(1+x)^2} + \sqrt[3]{(1-x)^2} - \sqrt[3]{1-x^2} = 1 \Rightarrow a^2 + b^2 - ab = 1$$

$$\begin{cases} a^3 + b^3 = 2 \\ a^2 + b^2 - ab = 1 \end{cases}$$

Denote:

$$\begin{cases} S = a + b \\ P = ab \end{cases} \Rightarrow \begin{cases} S^3 - 3SP = 2 \\ S^2 - 2P - P = 1 \end{cases} \Rightarrow \begin{cases} S(S^2 - 3P) = 2 \\ S^2 - 3P = 1 \end{cases} \Rightarrow S = 2$$

$$S^2 - 3P = 1 \Rightarrow 4 - 3P = 1 \Rightarrow P = 1$$

$$\begin{cases} S = 2 \\ P = 1 \end{cases} \Rightarrow a = b = 1 \Rightarrow \begin{cases} \sqrt[3]{1+x} = 1 \\ \sqrt[3]{1-x} = 1 \end{cases} \Rightarrow x = 0$$