

ROMANIAN MATHEMATICAL MAGAZINE

Solve for real numbers:

$$\sqrt[3]{1+x} - \sqrt[3]{1-x} + \sqrt[3]{1-x^2} = 1$$

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Denote:

$$\begin{cases} a = \sqrt[3]{1+x} \\ b = \sqrt[3]{1-x} \end{cases} \Rightarrow \begin{cases} 1+x = a^3 \\ 1-x = b^3 \end{cases} \Rightarrow a^3 + b^3 = 2$$

$$\sqrt[3]{1+x} - \sqrt[3]{1-x} + \sqrt[3]{1-x^2} = 1 \Rightarrow a - b + ab = 1$$

$$a(1+b) - (1+b) = 0 \Rightarrow (a-1)(b+1) = 0$$

$$a-1=0 \Rightarrow a=1 \Rightarrow 1^3 + b^3 = 2 \Rightarrow b=1$$

$$\Rightarrow \begin{cases} \sqrt[3]{1+x} = 1 \\ \sqrt[3]{1-x} = 1 \end{cases} \Rightarrow \begin{cases} 1+x = 1 \\ 1-x = 1 \end{cases} \Rightarrow x = 0$$

$$b+1=0 \Rightarrow b=-1 \Rightarrow a^3 = 3 \Rightarrow a = \sqrt[3]{3}$$

$$\sqrt[3]{1-x} = -1 \Rightarrow 1-x = -1 \Rightarrow x = 2 \Rightarrow S = \{0, 2\}$$